



A First  
Course In  
Atmospheric  
Radiation

Second Edition

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**Atmospheric Radiation**  
Second Edition

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